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Supershell Effect and Stability of Classical Periodic Orbits in Reflection-Asymmetric Superdeformed Oscillator

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Abstract

A semiclassical analysis is made of the origin of an undulating pattern in the smoothed level density for a reflection-asymmetric superdeformed oscillator potential. It is suggested that, when the octupole-type deformation increases, an interference effect between two families of periodic orbit with the ratio of periods approximately 2:1 becomes stronger and thus a pronounced "supershell" structure appears.

The quantum-energy spectrum in the axially-symmetric oscillator potential with frequency ratio $\omega_\perp/\omega_z$=2 (called "superdeformed" oscillator) is known to have "supershell" structure (*i.e.*, modulation with periodicity $2\hbar\omega_{\rm sh}$ in the oscillating level density, $\omega_{\rm sh}$ being the basic frequency of the superdeformed oscillator). We have indicated in our previous paper[1)] that the supershell effect is significantly enhanced when octupole (Y_{30}) deformation is added to the 2:1 deformed harmonic-oscillator potential, and suggested that this enhancement might be responsible for the odd-even effect (with respect to the shell quantum number $N_{\rm sh}$) in stability of superdeformed states against octupole deformed shape, discussed by Bengtsson et al.,[2)] Höller and Åberg,[3)] and Nazarewicz and Dobaczewski.[4)]

The single-particle Hamiltonian we used in our analysis is

$$h = \frac{\boldsymbol{p}^2}{2M} + \sum_{i=x,y,z} \frac{M\omega_i^2 x_i^2}{2} - \lambda_{3K} M\omega_0^2 \left(r^2\, Y_{3K}(\Omega)\right)'' , \tag{1}$$

where $\omega_x = \omega_y = 2\omega_z \equiv 2\omega_{\rm sh}$ and $\omega_0^3 = \omega_x\omega_y\omega_z$. The double primes denote that the variables in parenthesis are defined in terms of the doubly-stretched coordinates[5] $x_i'' = (\omega_i/\omega_0)\, x_i$. In the following, we limit to the case $K = 0$. When the variables are scale-transformed to dimensionless ones, the Hamiltonian (1) is rewritten as

$$h = \frac{\boldsymbol{p}^2}{2} + \left(\frac{r^2}{2} - \lambda_{30}\, r^2\, Y_{30}\,(\theta)\right)'' , \tag{2}$$

where $x''=2x$, $y''=2y$ and $z''=z$.

As is well known, the quantum level density $g\,(E) = \sum_n \delta\,(E - E_n)$ is expressed in semiclassical theory as[6]

$$g\,(E) \simeq \bar{g}\,(E) + \sum_{\gamma} A_\gamma(E) \cos\left(S_\gamma(E)/\hbar - ({\rm phases})_\gamma\right) , \tag{3}$$

where $\bar{g}$ is an average level density corresponding to the Thomas-Fermi approximation, and the second term in the r.h.s. represents the oscillatory contributions from periodic orbits, S_γ is the action integral $\oint_\gamma \boldsymbol{p} \cdot d\boldsymbol{q}$, and the amplitude factor A_γ is mainly related to the stability of the orbit γ. When one is interested in an undulating pattern in $g\,(E)$ smoothed to a finite resolution δE (*i.e.*, shell structure), it is sufficient to only consider short periodic orbits with the periods $T_\gamma < 2\pi\hbar/\delta E$. The supershell structure is expected to arise from interference effects between orbits with different periods T_γ. The short periodic orbits are calculated by Monodromy Method[7] and shown in Fig. 1.

Fig. 1

For the Hamiltonian system under consideration whose phase space is constructed with both regular and chaotic regions, evaluation of A_γ in eq. (3) is not always easy because

the stationary-phase approximation breaks down near resonances which take places rather frequently in the regular regions. Fortunately, however, by virtue of the scaling property, $h(\alpha\boldsymbol{p}, \alpha\boldsymbol{q}) = \alpha^2 h(\boldsymbol{p}, \boldsymbol{q})$, we can use the Fourier-transformation techniques[8] and extract informations about classical periodic orbits from quantum energy spectrum. The scaling rules for variables in eq. (3) are[9]

$$\begin{aligned} \bar{g}(E) &= E^2 \bar{g}(1), \\ S_\gamma(E) &= E\,T_\gamma, \\ A_\gamma(E) &= E^{k_\gamma} A_\gamma(1) \qquad \begin{cases} k_\gamma = 0 & \text{for isolated orbits,} \\ k_\gamma = \frac{1}{2} & \text{otherwise.} \end{cases} \end{aligned} \tag{4}$$

The last equality is obtained under the stationary-phase approximation. Using these relations, it is easy to see that the Fourier transform of eq. (3) multiplied by an appropriate weighting factor E^{-k} will exhibit peaks at the periods T_γ of classical periodic orbits and the heights of the peaks represent the strengths of their contributions. In Fig. 2, we show the power spectrum $P(s)$ for several values of λ_{30}, taking $k = \frac{1}{2}$ appropriate to non-isolated orbits;

$$P(s) = \left| \sum_n \frac{e^{isE_n}}{\sqrt{E_n}} \right| . \tag{5}$$

Fig. 2

We see nice correspondence between peak locations of $P(s)$ and periods of classical periodic orbits. The most important observation is that relative intensity between peaks at $s \approx \pi$ and $s \approx 2\pi$ changes as the octupole deformation parameter λ_{30} increases. This result indicates that the enhancement of the supershell effect (shown in Fig. 6 of Ref. 1)) may be explained as due to the growth of the interference effect between classical periodic orbits with periods $T \approx \pi$ and those with $T \approx 2\pi$.

To understand the cause of the change in relative intensity mentioned above, let us investigate properties of the classical periodic orbits. Calculating periodic orbits by the

Monodromy Method,[7] we obtain stability matrices M_γ for the periodic orbits γ. They are linearized Poincaré maps at the periodic orbits defined as

$$\begin{pmatrix} \delta\boldsymbol{p}\,(T_\gamma) \\ \delta\boldsymbol{q}\,(T_\gamma) \end{pmatrix} = M_\gamma \begin{pmatrix} \delta\boldsymbol{p}\,(0) \\ \delta\boldsymbol{q}\,(0) \end{pmatrix} + \mathcal{O}\,(\delta^2), \tag{6}$$

where $(\delta\boldsymbol{p}, \delta\boldsymbol{q})$ represent deviations from the periodic orbits γ in phase space. These six-dimensional matrices M_γ are real and symplectic, so that eigenvalues of each M_γ appear in pairs $\pm(e^\alpha, e^{-\alpha})$, α being real or pure imaginary. As the Hamiltonian (2) is axially-symmetric, classical orbits are usually non-isolated. For such orbits, each stability matrix has 4 unit eigenvalues. Values of $\mathrm{Tr}\,M$ written in Fig. 1 are sums of the remaining 2 eigenvalues which determine stabilities of the periodic orbits; $\alpha{=}iv$ is pure imaginary and $|\,\mathrm{Tr}\,M| = |2\cos v| \le 2$ when the orbit is stable, while $\alpha{=}u$ is real and $|\,\mathrm{Tr}\,M| = |\pm 2\cosh u| > 2$ when the orbit is unstable. Under the stationary-phase approximation, the amplitude factors A_γ in eq. (3) are inversely proportional to $\sqrt{|\,\mathrm{Tr}\,M_\gamma - 2|}$. Fig. 3 shows values of $\mathrm{Tr}\,M$ for relevant orbits calculated as functions of the octupole-deformation parameter λ_{30}.

Fig. 3

From this figure, we see that the orbits with $T \approx \pi$ are always stable and their values of $\mathrm{Tr}\,M$ approach to 2 with increasing λ_{30}, while the orbit B (C,C') with $T \approx 2\pi$ become unstable (more unstable). Therefore, we can expect that the contributions from orbits with $T \approx \pi$ increase when λ_{30} becomes large, while those from orbits with $T \approx 2\pi$ decrease. This result suggests that the enhancement of the supershell effect stems from the difference of the stability against octupole deformation between these two families of periodic orbit.

A more detailed analysis of the supershell structure in reflection-asymmetric superdeformed oscillator potentials will be given in a forthcoming full-length paper.[9] The author thanks Prof. Matsuyanagi for carefully reading the manuscript.

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Fig. 1. Short periodic orbits for the Hamiltonian (2) with $\lambda_{30} = 0.4$. Upper part: Planar orbits in the plane containing the symmetric axis z. Lower part: A circular orbit in the plane perpendicular to the symmetry axis (A') and a three-dimensional orbit (C'). Their projections on the (x, y) plane and on the (z, y) plane are shown.

Fig. 2. Power spectra $P(s)$ defined by eq. (5) for $\lambda_{30} = 0.2 \sim 0.4$. The summation is taken up to n=200. Arrows indicate periods of the classical periodic orbits (see Fig. 1) and their repetitions.

Fig. 3. Traces of the stability matrices M for the periodic orbits shown in Fig. 1 (see text for their definitions). For the isolated orbit A', the stability matrix M has 2 unit eigenvalues and the remaining 4 eigenvalues appear in two pairs $(e^{\alpha_a}, e^{-\alpha_a})$ and $(e^{\alpha_b}, e^{-\alpha_b})$. Thus, A'$_a$ and A'$_b$ denote the traces of these pairs, respectively.